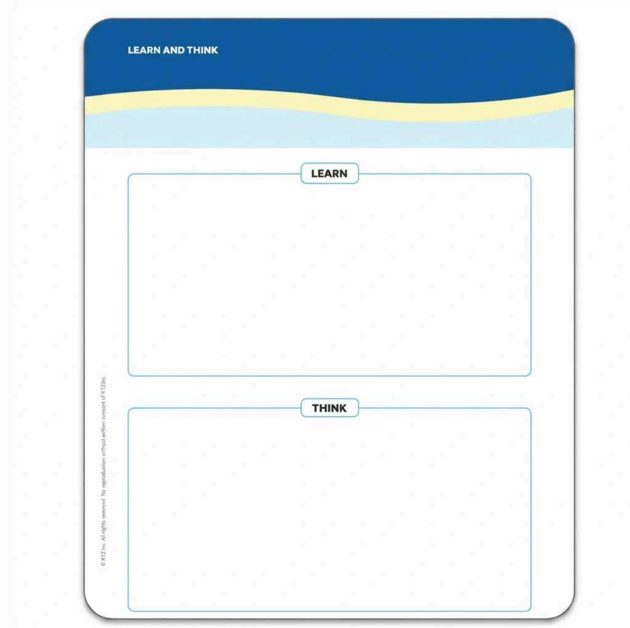
LEARN

Imagine a social media post spreading like wildfire, growing every single second without pause. This kind of perfectly smooth, continuous growth can't be modeled by a simple doubling formula. It needs a constant built for **continuous change**.

That constant is *e*, and it's one of the most important numbers in mathematics. You can use the *Learn* part of the page in the workbook to show your thinking or take notes.



Select each tab to know more about *e* and natural logarithm.

Meet e | Natural Logarithm

The Constant e

The number e is an **irrational constant**. It cannot be written as a simple fraction and its decimal digits never repeat.

$$e \approx 2.71828182845\ldots$$

Unlike π, which describes circles, e emerges from the mathematics of **continuous growth**. Any time a quantity grows at every instant (not in steps), e is the natural base to use.

Meet e | Natural Logarithm

What Is ln(*x*)?

The natural logarithm is the **inverse** of e^x. It answers the question:

What power must e be raised to in order to get x?

It is written $\ln(x)$ and is defined as:

$$\ln(x) = \log_e(x)$$

So $\ln(e^3) = 3$, because 3 is the exponent you need on e to produce e^3.

Note: $\ln$ is not a variable. It is an *operation*, just like $\log$ or $\sqrt{\ }$.

Identifying the Inverse Operations

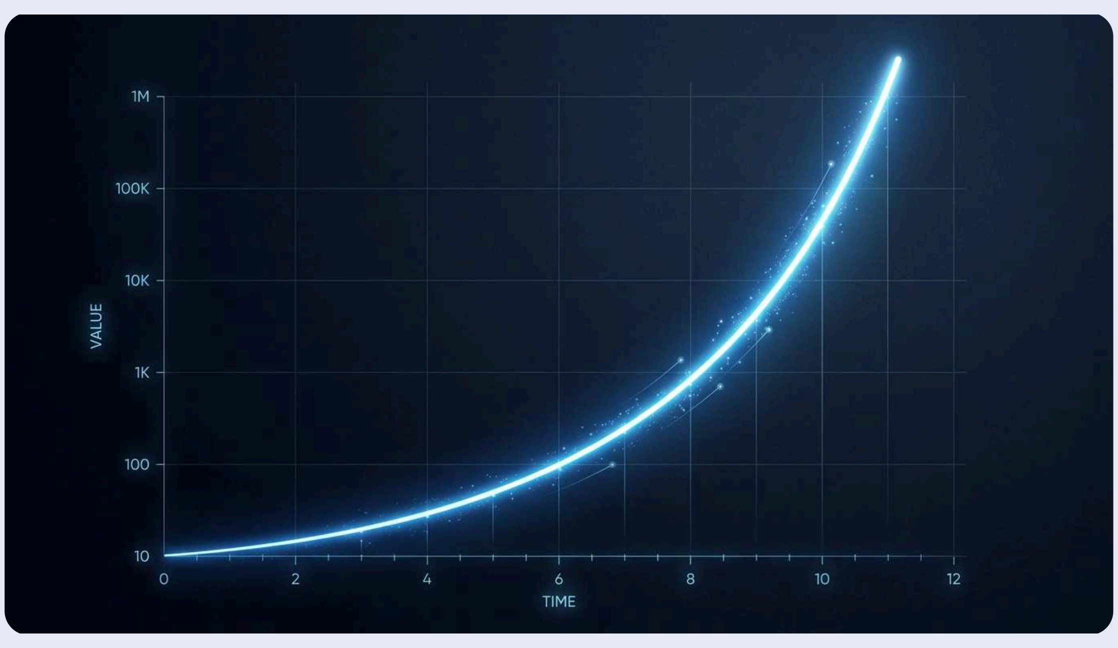
Work through each step to understand how e^x and $\ln(x)$ are connected, and how that connection lets you solve continuous growth equations.

Select through the screens to Identify the Inverse Operations.

1 Meet the Natural Base

You've encountered growth models that use a base of 2 or 10, but what base does *continuous* growth require?

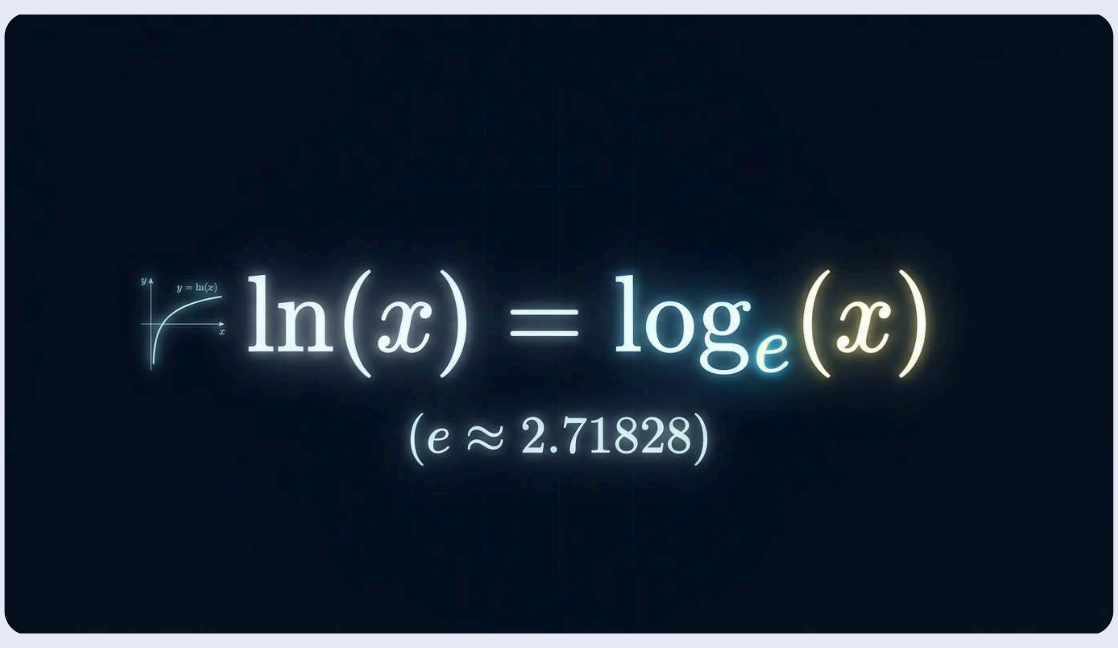
The answer: The constant $e \approx 2.718$ appears whenever quantities grow without interruption. The function $y = e^x$ models this constant, unbroken growth.



2 Unlock the Secret Logarithm

If e^x models continuous growth, what operation would you use to *undo* it and isolate an unknown exponent?

The answer: The natural logarithm, written $\ln(x)$. It is equivalent to $\log_e(x)$ (a logarithm with base e), and it is the exact inverse needed to reverse continuous growth equations.



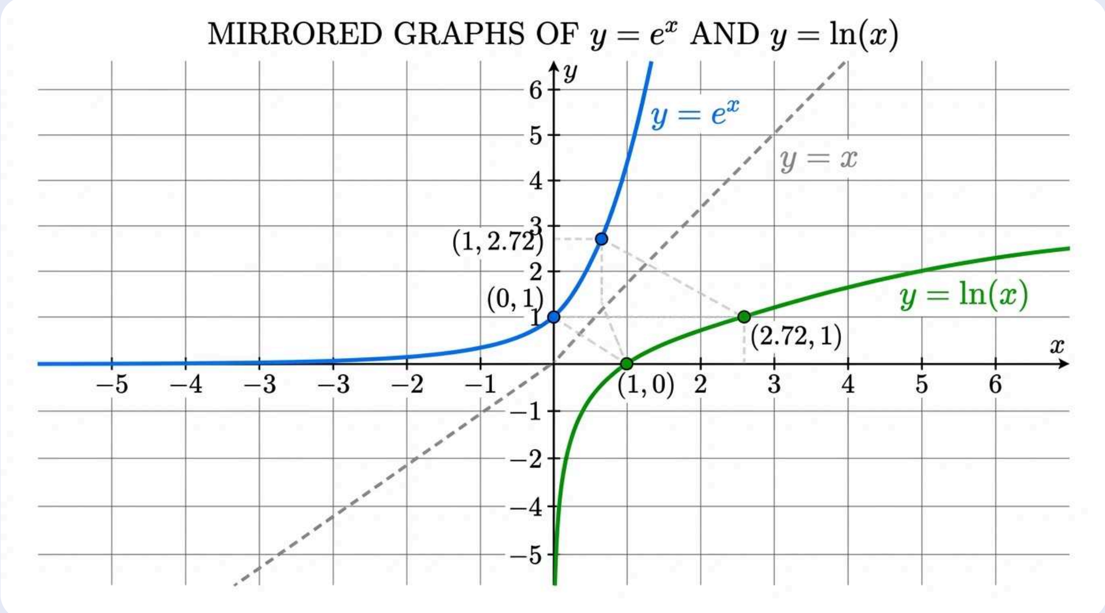
3 Connect the Inverse Loop

When e^x and $\ln(x)$ are applied one after the other, they cancel completely. This is the **cancellation property**:

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

For example: start with $x = 3$. Apply e^x and get $e^3 \approx 20.09$. Apply $\ln$ to the result: $\ln(20.09) = 3$. You are back where you started.



4 Convert with Confidence

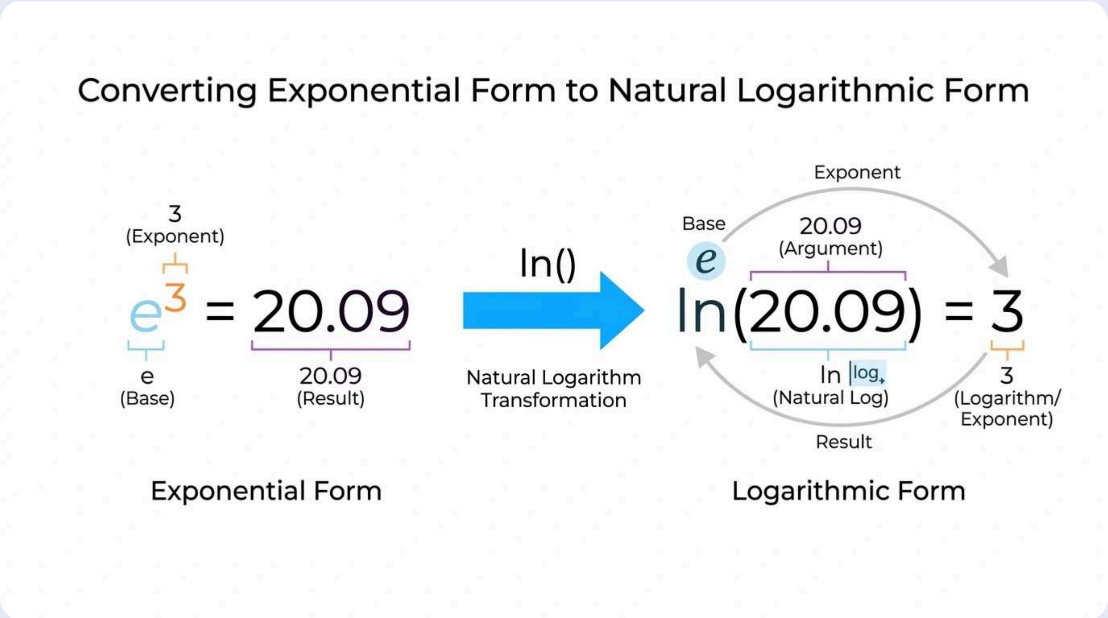
The conversion rule is:

$$e^x = y \leftrightarrow \ln(y) = x$$

Worked example: In $e^3 = 20.09$, the result (20.09) moves inside $\ln$ and the exponent (3) becomes the output:

$$e^3 = 20.09 \Rightarrow \ln(20.09) = 3$$

Natural logarithms produce exact solutions. Exponential equations can also be solved graphically or with a table of values, but $\ln$ is the standard method when an exact answer is needed.



Before moving to the practice cards, try this on your own. Which is the correct logarithmic form of $e^5 = 148.41$?

Select one

1. $\ln(5) = 148.41$

   In $e^5 = 148.41$, the number 5 is the exponent and 148.41 is the result. The rule $e^x = y \leftrightarrow \ln(y) = x$ puts the result inside $\ln$ and the exponent becomes the output: $\ln(148.41) = 5$.

2. $e^{148.41} = 5$

   This reverses the form but does not use $\ln$ notation. The logarithmic form requires the $\ln$ operator. Using $e^x = y \leftrightarrow \ln(y) = x$: $\ln(148.41) = 5$.

3. $\ln(148.41) = 5$

   The result (148.41) moves inside $\ln$ and the exponent (5) becomes the output, following the rule $e^x = y \leftrightarrow \ln(y) = x$.

4. $5 \cdot \ln(e) = 148.41$

   This misapplies the conversion. The rule $e^x = y \leftrightarrow \ln(y) = x$ does not involve multiplying. The correct logarithmic form is $\ln(148.41) = 5$.

Common Misconception: $\ln$ is **not** a variable. It is the natural logarithm operation (log base e). This means you cannot divide both sides by $\ln$ or treat it as a number.

Correct approach for $e^x = 20$: Take the natural log of both sides → $\ln(e^x) = \ln(20)$ → $x = \ln(20)$.

Select each flipcard to learn more.

Rewrite in logarithmic form:

$e^{2t} = 100$

$\ln(100) = 2t$

The entire exponent ($2t$) stays together as a unit and becomes the output. The result (100) moves inside $\ln$.

Rewrite in exponential form:

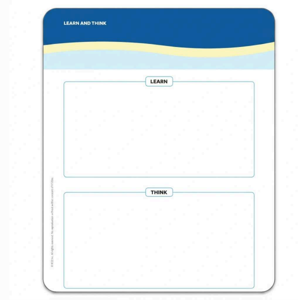
$\ln(7) = p$

$e^p = 7$

The output of $\ln$ becomes the exponent on e. The input (7) becomes the result.

THINK

Test what you've learned about e, natural logarithms, and form conversion. You can use the *Think* part of the page in the workbook to show your thinking or take notes.



Apply the cancellation property.

What is the value of $e^{\ln 7}$?

Select one

1. $\ln(7)$

   $\ln(7)$ is the input to the exponent, not the result. The cancellation property states $e^{\ln x} = x$—the two operations undo each other.

2. $7e$

   $7e$ would mean multiplying 7 by e. When e^x and $\ln(x)$ are composed, they cancel completely—they do not multiply.

3. 7

   By the cancellation property, $e^{\ln x} = x$ for any $x > 0$. So $e^{\ln 7} = 7$. The two inverse functions undo each other.

4. $\frac{1}{7}$

   Raising e to the power $\ln(7)$ is not the same as taking a reciprocal. The cancellation property gives $e^{\ln x} = x$, so the result is 7.

Which option is the natural logarithmic form of $e^k = 30$?

Select one

1. $\ln(k) = 30$

   This swaps the roles of k and 30. In $e^k = 30$, the exponent is k and the result is 30. Using the rule $e^x = y \leftrightarrow \ln(y) = x$: the result (30) goes inside $\ln$, and the exponent (k) becomes the output.

2. $\ln(30) = k$

   Using the rule $e^x = y \leftrightarrow \ln(y) = x$: the base e drops away, the result 30 moves inside $\ln$, and the exponent k becomes the output.

3. $k = e^{30}$

   This rewrites in exponential form, not logarithmic form. To get the logarithmic form, apply $\ln$ to both sides: $\ln(e^k) = \ln(30)$, giving $k = \ln(30)$.

4. $\log(k) = 30$

   $\log(k)$ represents a common logarithm (base 10), not a natural logarithm. When the base is e, the correct notation is $\ln$, not $\log$.

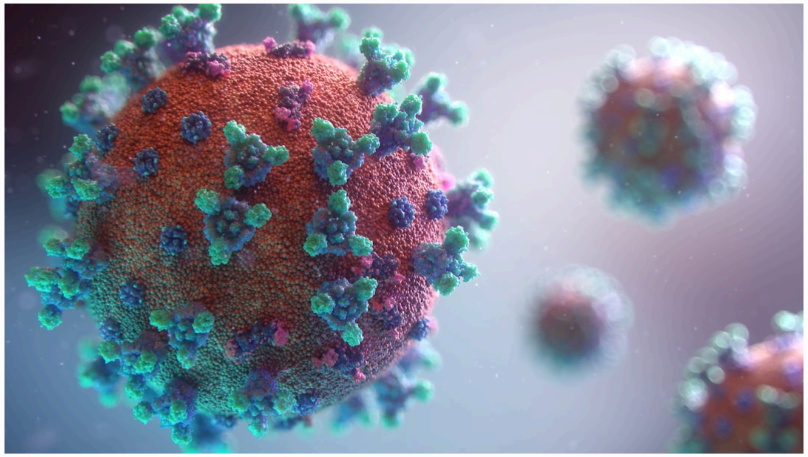
A student is analyzing a continuous growth model for a streaming video. The scenario is represented by the equation $e^x = 55$. The student claims that to find the value of x, they must divide 55 by 2.718.

Explain why the student's method is incorrect, state the correct logarithmic form needed to solve for x, and use technology to evaluate your answer to the nearest thousandth.

Type answer

In the continuous growth model $A = Pe^{rt}$: A is the final amount, P is the initial amount, r is the continuous growth rate, and t is time.

Use everything you have practiced: set up the equation, isolate the exponential, convert to logarithmic form, use technology to evaluate, and interpret your answer.



A population of bacteria grows from **500** to **1500** organisms in **4 hours**, following the model $A = Pe^{rt}$.

Substitute the known values, isolate the exponential expression, convert to natural logarithmic form, then use technology to evaluate $\ln(3)$ and solve for r. Explain what r represents in context. **Show all steps** and express r as a decimal rounded to three places.

Type answer